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APPENDIX B

SYNOPSIS OF THE MONTGOMERY MODEL

Consider a geographical region or airshed in which there are k stationary sources of a single air pollutant. Of these, $n \leq k$ are producers of a single product (say, electricity) who may behave strategically toward each other in their common output market. Distributed about the region are m receptor sites. Environmental quality in the region is defined and regulated according to the level of pollutant concentration at each of these sites. Associated with each firm i is a vector $d_i = (d_{i1},\ldots,d_{im})$ which specifies the dispersion of pollutant to each of the m sites that results when firm i emits one unit of the pollutant.

Suppose the environmental authority specifies $Q^* = (q^*_1,\ldots,q^*_m)$, a vector of the maximum allowable pollutant concentrations at each receptor. Let the yearly emissions level of firm i be given by e_i; let $E = (e_1,\ldots,e_k)$. Then if we let $D = [d_1,\ldots,d_k]$ be an $m \times k$ matrix of dispersion coefficients, the quality standards are met if $ED \leq Q^*$, where vectors are understood to be conformable and oriented so that the given operations are well defined. This constraint on the system may be applied to the planner's problem; any potential trading solution that violates it is not allowed.

Montgomery's[1] formulation is similar to this one. He proceeds by exploiting the perfect competition assumption, using an envelope result to reduce the emitter's problem to one of minimizing the cost of reducing emissions to the maximum allowed level. If emitters always choose output production levels optimally for a given e_i, and if the price they receive for the output is fixed, the objective function of a polluting firm may be written as an implicit cost function. The firm chooses its output level so as to minimize this cost, and in doing so it also maximizes its profits. Write the cost of abating pollution emissions from the optimal unconstrained level to the legally specified level as $C_i(e_i)$, where, again, the cost of production is suppressed. Then the regulatory authority, in choosing a CAC system that specifies e_i for each

[1] David W. Montgomery, "Market in Licenses and Efficient Pollution Control Programs," *Journal of Economic Theory* (1972): 395-418.

firm, minimizes the cost of achieving air quality Q^* by solving the following program

$$\min_{e_1,\ldots,e_k} \sum_{i=1} C_i(e_i) \quad \text{(1)}$$

$$\text{s.t.} \quad E \geq Q$$

$$0 \leq E$$

The solution to this program, as demonstrated by Montgomery and others, has desirable efficiency properties under suitable conditions.

The formal mathematical models that underlie this logic can be presented in a fairly simple form. Here, a numerical example that gives the flavor of the argument should prove helpful. It should be kept in mind that this version is quite simple. Nevertheless, the logic is the same as in the more elaborate versions that are to be found in the literature. It is also akin to those versions in the way that it relies upon a set of simplifying assumptions that makes the market perfect in the economist's sense: everyone knows everything, nobody has any market power, and so on.

Consider a world in which two coal-burning plant supply all of the electricity. These plants (call them plant 1 and plant 2) are located near each other, and each emits a certain amount of sulfur dioxide into the air for every unit of coal it burns. Imagine that the world is now in equilibrium, that together these plants exactly meet the demand for electricity, that they know everything that can be known about each other, about tomorrow's weather, and so on. Suppose also that utility regulation has not yet arrived, so that neither plant is concerned with whether its decisions will be approved in an upcoming hearing. Finally, suppose that plant 1 is currently emitting 100 tons of SO_2 annually, and that plant 2 is emitting 150 tons (for a total level of emissions of 250 tons). We must make another assumption about the plants' behavior: for a given level of electricity generation, and for a given level of sulfur emissions, each plant gets everything else right. That is, all of the usual optimizing behavior (employing the right number of people, burning the optimal amount of coal, building a plant of exactly the right size, and so on) is taking place. No mistakes are being made anywhere. This assumption is critical because it allows us to put all but one of the plant's decisions in the background, and to write an *abatement cost function* that purports to represent everything interesting about the plant's operations. This function gives the cost of doing business, but in such a way that cost depends only upon the level of SO_2 emissions. What's more, this function is such that for a given level of generation, costs will *increase* as emissions *decrease*. This is so because in order to produce at the same level as emissions go down, more must be spent on abatement equipment, and so on.

In this example, plant 1 is a relatively new unit, so that its level of emissions is lower than that of plant 2, and it is also cheaper for plant 1 to abate. Let's say that sulfur emissions of plant 1 are represented by e_1, and those of plant 2 by e_2. The abatement cost functions for the two plants are assumed to be given by

$$C_1(e_1) = \frac{5{,}000}{\sqrt{e_1}}, \quad C_2(e_2) = \frac{40{,}000}{\sqrt{e_2}}.$$

In the world as it exists here, then, with plant 1 emitting 100 tons ($e_1 = 100$) at a cost of \$500 and with plant 2 emitting 150 tons at a cost of \$3,266, total costs for the industry equal \$3,766.

The example includes one additional actor: an environmental regulator. This benevolent government employee, charged with protecting the environment, decides that the annual level of emissions should be reduced by 40 percent to 150 tons. This number is made the law of the land, and the regulator is charged with devising a plan for meeting the new environmental objective. One of two alternatives for achieving the required 100 tons of abatement may be selected.

The first is a simple version of a command-and-control regime: each plant will be required to cut back in proportion to its initial pollution level. This is the proportional reduction (PR) plan. The second is to implement a marketable pollution permit scheme, whereby the two plants are given a total of 150 allowances (how these are divided between them may or may not concern our regulator), each granting its holder the right to emit a ton of sulfur dioxide. This is the tradable allowance (TA) plan. Under TA it is illegal to emit more sulfur than represented by the allowances a plant holds. With this program the two plants have the freedom to reach an agreement among themselves---free, in particular, from further government intervention---about how much each plant should pollute. Whatever the initial allocation of allowances, the two plants buy and sell allowances from one another so that each owns exactly enough to emit according to its optimal plan.

Now the setup is complete. The regulatory decision about which plan to implement is based only upon total cost considerations. Whichever plan is cheaper for the industry will be chosen. The results of the relevant calculations appear in Table B-1. Without pollution regulation, the numbers are as above (total cost equals $3,766). These appear in the first column of the table. The PR plan is easy to implement, and requires very little in the way of calculation. Each plant must come up with a reduction of 40 percent, so that plant 1 winds up emitting $e_1 = 60$ tons, and plant 2 emits $e_2 = 90$ tons. The corresponding costs are C_1 = $645.50 and C_2 = $4,216.40 (recall that costs go up as emission levels fall). The total cost is $4,861.90.

Under the TA plan, each plant sets the marginal cost of abatement equal to the allowance price. Trade between the two plants will occur in such a way as to equalize this marginal cost across the plants. In order to decide which plan to implement, the regulator will want to calculate the optimal decision under this plan, and then to compare it to $4,861.90. This involves minimizing the total cost of compliance (equalling the sum of the two cost functions), given that total emissions cannot exceed 150 tons. The cost-minimizing decision is for plant 1 to reduce the most, emitting a total of only 30

TABLE B-1

TOTAL OPERATING COSTS UNDER VARIOUS POLLUTION CONTROL REGIMES

	Status Quo	Proportional Reductions	Tradable Coupons
Plant 1 Emissions	100	60	30
Plant 2 Emissions	150	90	120
Total Emissions	250	150	150
Plant 1 Costs	$ 500	$ 645.50	$ 912.87
Plant 2 Costs	3,266	4,216.40	3,651.48
Total Costs	3,766	4,861.90	4,564.35
Coupon Price	n.a.	n.a.	$15.21

tons, and for plant 2 to emit 120 tons. The corresponding costs of operating (ignoring the purchase or sale of allowances) are $912.87 for plant 1 and $3,651.48 for plant 2. Total cost to the industry is $4,564.35. It is also relatively easy to calculate the market-clearing allowance price. This price, equaling the marginal abatement cost for both plants, will equal $15.21.

It is easy to see that the tradable allowance plan should be selected. Under this plan, total cost of compliance with the environmental standard is $297.55 less than under the proportional reduction plan. It is essential, in order fully to understand this example, to keep in mind exactly what goes wrong if the PR plan is implemented. Plant 2 abates at a relatively high cost, which means the resources devoted to pollution control when this plant is emitting only 90 tons are not used wisely. The same level of expenditures on abatement at plant 1 would have purchased a greater level of abatement. This is the source of the inefficiency and of the additional cost of the PR plan over the TA plan.